

Information about Information
about Cosmology from Large-Scale
Structure

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Information about Information about Information about Cosmology from Large-Scale Structure

- **Cosmological Fisher information in the matter power spectrum as a function of scale from N -body simulations**
- **Same thing from the halo model; physical explanation**
- **Maybe difficult to extract cosmological information on non-linear scales from matter power spectrum, even under ideal circumstances.**
- **Other statistics useful? No information in things => information in nothing?**

Collaborators

István Szapudi

Chris Rimes

Andrew Hamilton



Fisher Information

- How much information about the depth of a (non-moving) fish?
- Michael Palin takes many measurements, finds fractional variance Var
- $Info = 1/Var$
- What if he adds another depth meter?



Fisher information

- Twice the info with 2 meters? Some of the variance is covariance, e.g. from water conditions.
- Construct covariance matrix.

$$C_{ij} = \langle \Delta d_i \Delta d_j \rangle \quad C = \begin{pmatrix} Var & a \times Var \\ a \times Var & Var \end{pmatrix}$$

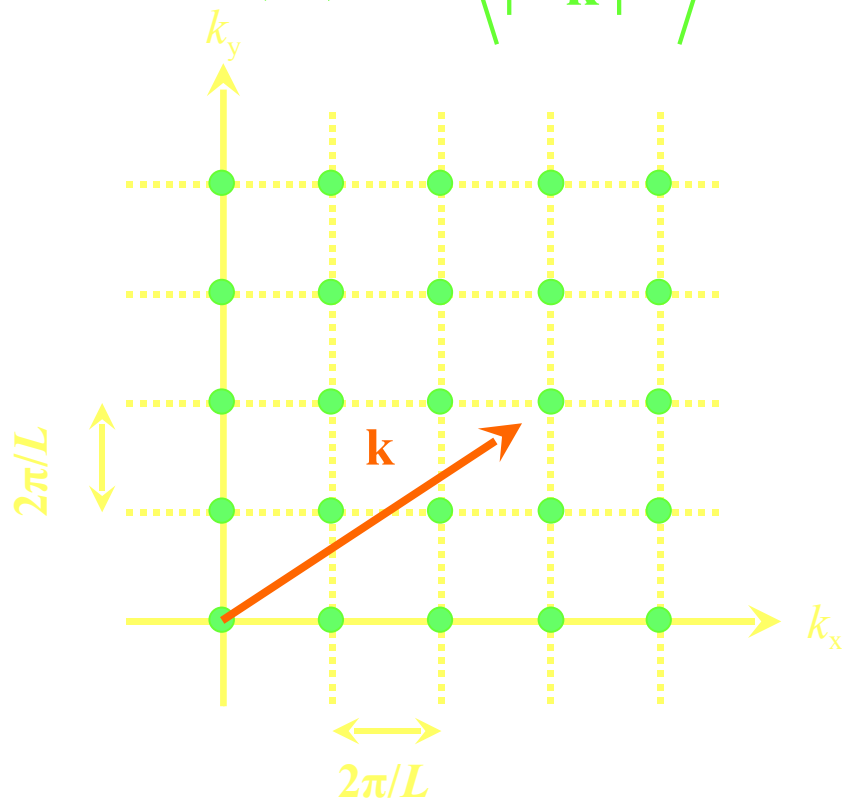
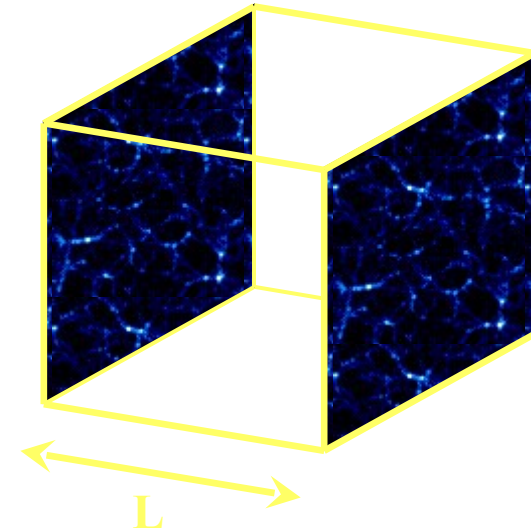
$$Info = \sum F_{ij} = \sum (C^{-1})_{ij} = \frac{2}{(1+a) \times Var}$$

- $a \rightarrow 1$: same info as one meter ($1/Var$)
- $a = 0$: double the info ($2/Var$)

The power spectrum

$$\delta(\mathbf{x}) \equiv \frac{\rho(\mathbf{x}) - \bar{\rho}}{\bar{\rho}} = \sum_{\mathbf{k}} \delta_{\mathbf{k}} e^{-i\mathbf{k}\cdot\mathbf{x}}$$

$$P(k) \equiv \left\langle |\delta_{\mathbf{k}}|^2 \right\rangle$$

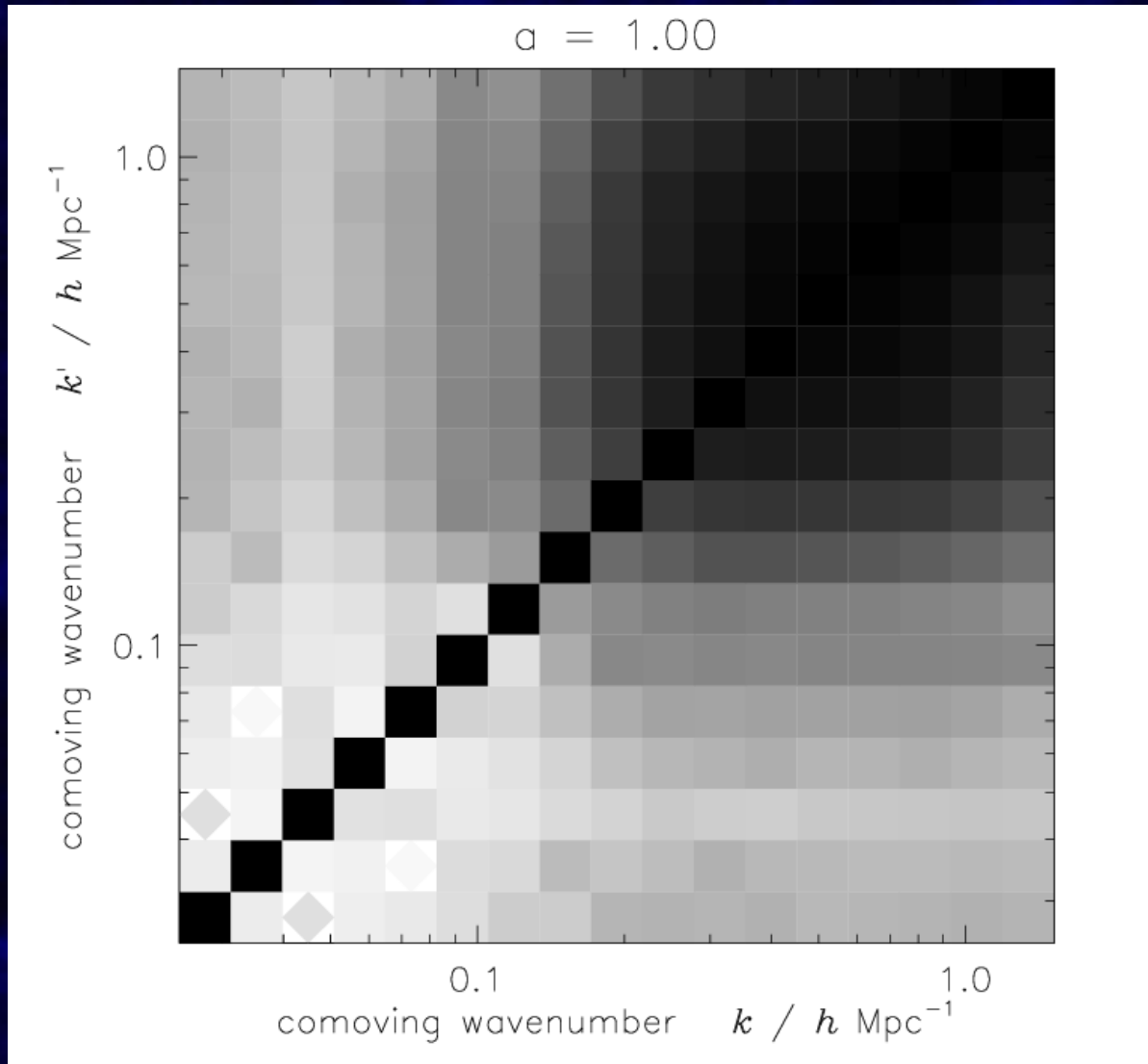


Gaussian field: all initial cosmological information in large-scale structure is in the power spectrum.

Does it stay there?

Growth of correlations

(Rimes & Hamilton)



$$\frac{C_{ij}}{\sqrt{C_{ii}C_{jj}}}$$

$$C_{ij} = \langle \Delta P_i \Delta P_j \rangle$$

400 realizations of $256 h^{-1} \text{ Mpc}$
box

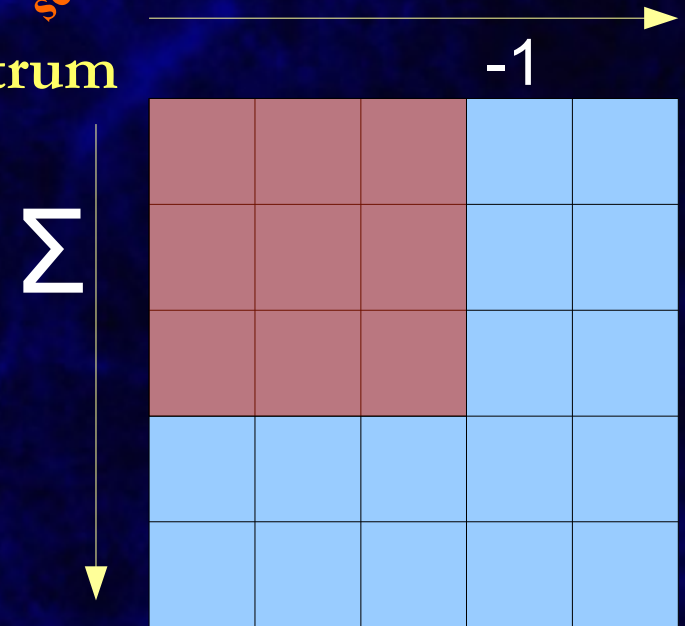
k increasing, scale *decreasing*

Fisher Information

To
smaller
scales

- The Fisher information in the power spectrum about parameters α and β : concavity of likelihood function

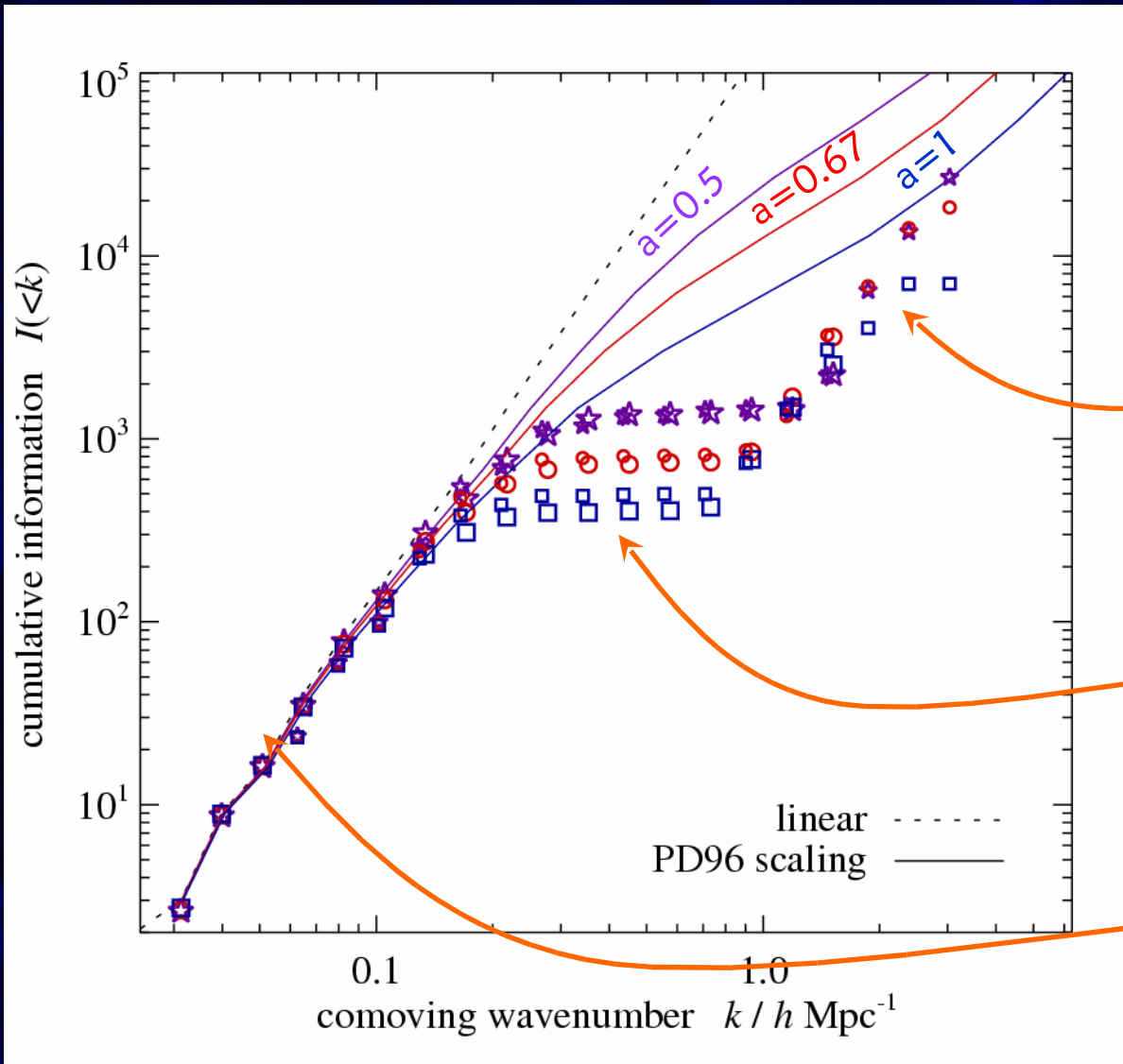
$$F_{\alpha\beta} \equiv - \left\langle \frac{\partial^2 \ln L}{\partial \alpha \partial \beta} \right\rangle_{ML}$$



- Cumulative information over a range of bins R :

$$F_{\alpha\beta}(R) = \sum_{i,j \in R} \frac{\partial \ln P_i}{\partial \alpha} (C_R^{-1})_{ij} \frac{\partial \ln P_j}{\partial \beta}$$

Cumulative information $I_A(<k)$



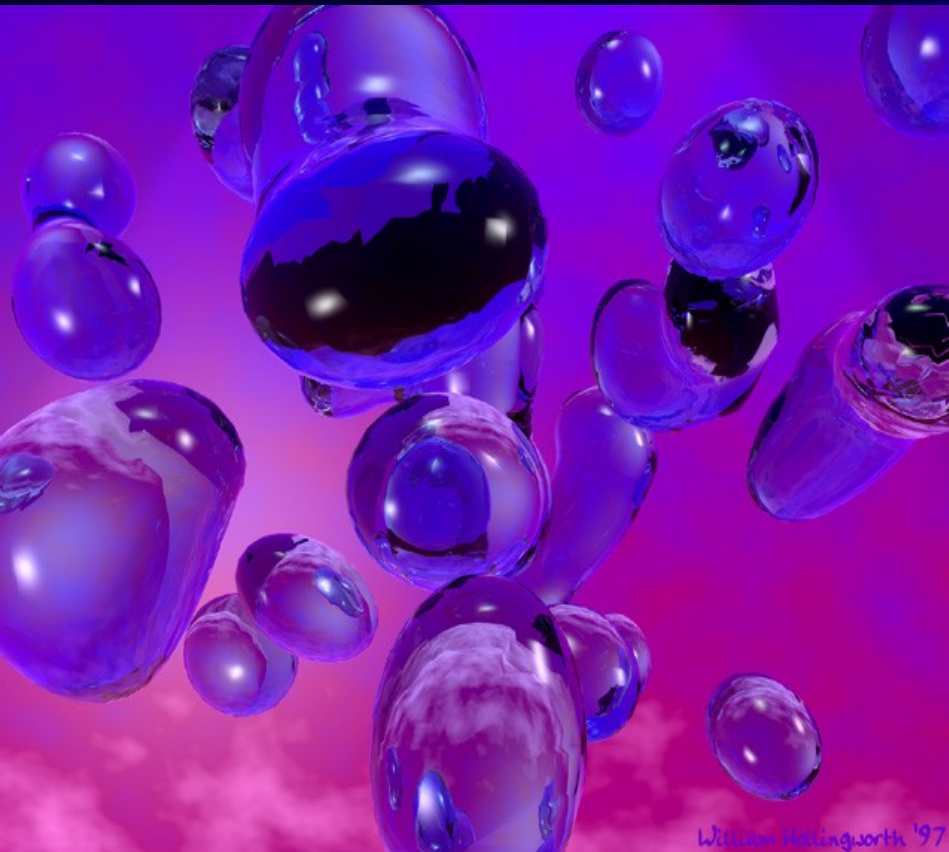
Information rises again at small scales (but more shallowly than k^3)

Very little additional info on “translinear” scales

$I_A(<k) \propto k^3$ in linear regime

Sharply demarcated behavior
in different regimes:

A job for the
halo model?



- Halo models: virialized dark-matter haloes distributed according to leading-order perturbation theory (not quite right – Smith, Scoccimarro & Sheth)
- Need 4-point function (trispectrum) to describe $\langle \Delta P_i \Delta P_j \rangle$, the product of two 2-point functions.

$$C_{ij} = \frac{1}{V_{survey}} \left[T_{ij} + \delta_{ij} (2\pi)^3 \frac{2P(k_i)^2}{V(k_i)} \right]$$

Non-Gaussian part

Gaussian part

where the trispectrum

$$T_{ij} = \left\langle T(k_i, -k_i, k_j, -k_j) \right\rangle_{k_i \cdot k_j}$$

Halo model trispectrum (Cooray & Hu 2001)

$$T = T^{1h} + T^{2h} + T^{3h} + T^{4h}$$

$$T_{ij}^{1h} = I_{ii}^{04}$$

$$T_{ij}^{2h} = \left\langle \left(P_i^{lin} I_i^{11} I_{ij}^{13} + 3 \text{ perm.} \right) + \left(P_{ij}^{lin} I_i^{11} I_j^{11} I_{ij}^{22} I_{ij}^{13} + 2 \text{ perm.} \right) \right\rangle$$

$$T_{ij}^{3h} = \left\langle \left[B^{pt}(k_i, -k_i, k_j, -k_j) I_{jj}^{12} + P_i^{lin} P_i^{lin} I_{jj}^{22} \right] I_i^{11} I_i^{11} + 5 \text{ perm.} \right\rangle$$

$$T_{ij}^{4h} = \left(I_i^{11} I_j^{11} \right)^2 \left\langle T^{pt}(k_i, -k_i, k_j, -k_j) \right\rangle + 2 \left(I_i^{11} I_j^{11} P_i^{lin} P_j^{lin} \right) \left(I_i^{21} I_j^{11} P_j^{lin} + I_j^{21} I_i^{11} P_i^{lin} \right)$$

I : halo profile integrals over mass

B^{pt} : Perturbation theory bispectrum

T^{pt} : Perturbation theory trispectrum

Behavior with **redshift**.

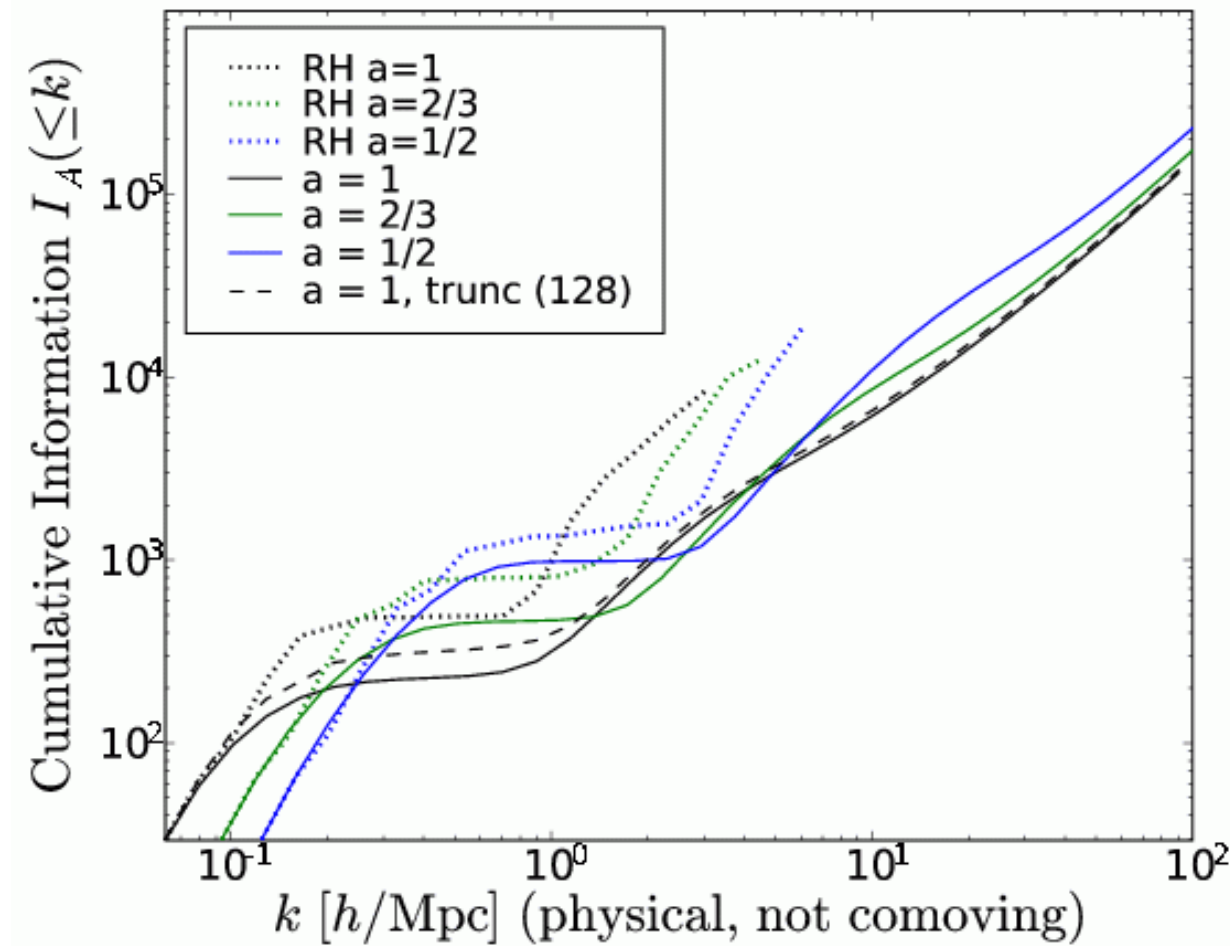
Expect info up to some small *physical* scale to:

- remain constant (\sim stable clustering?)

or

- decrease (info diverted into other statistics?)

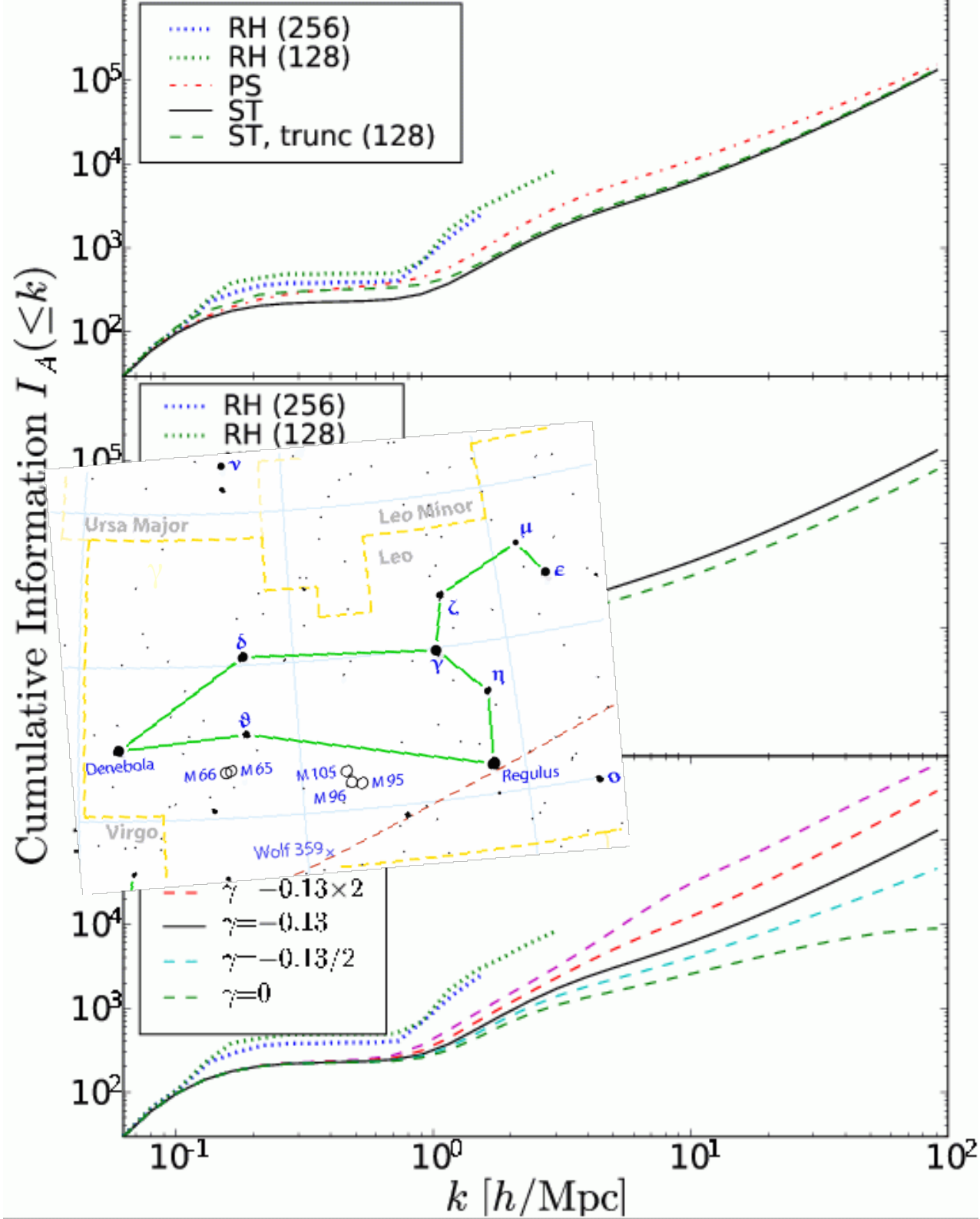
- constrain halo-model parameters this way?



Fiducial (Sheth-Tormen) model: information slowly decreases with time.

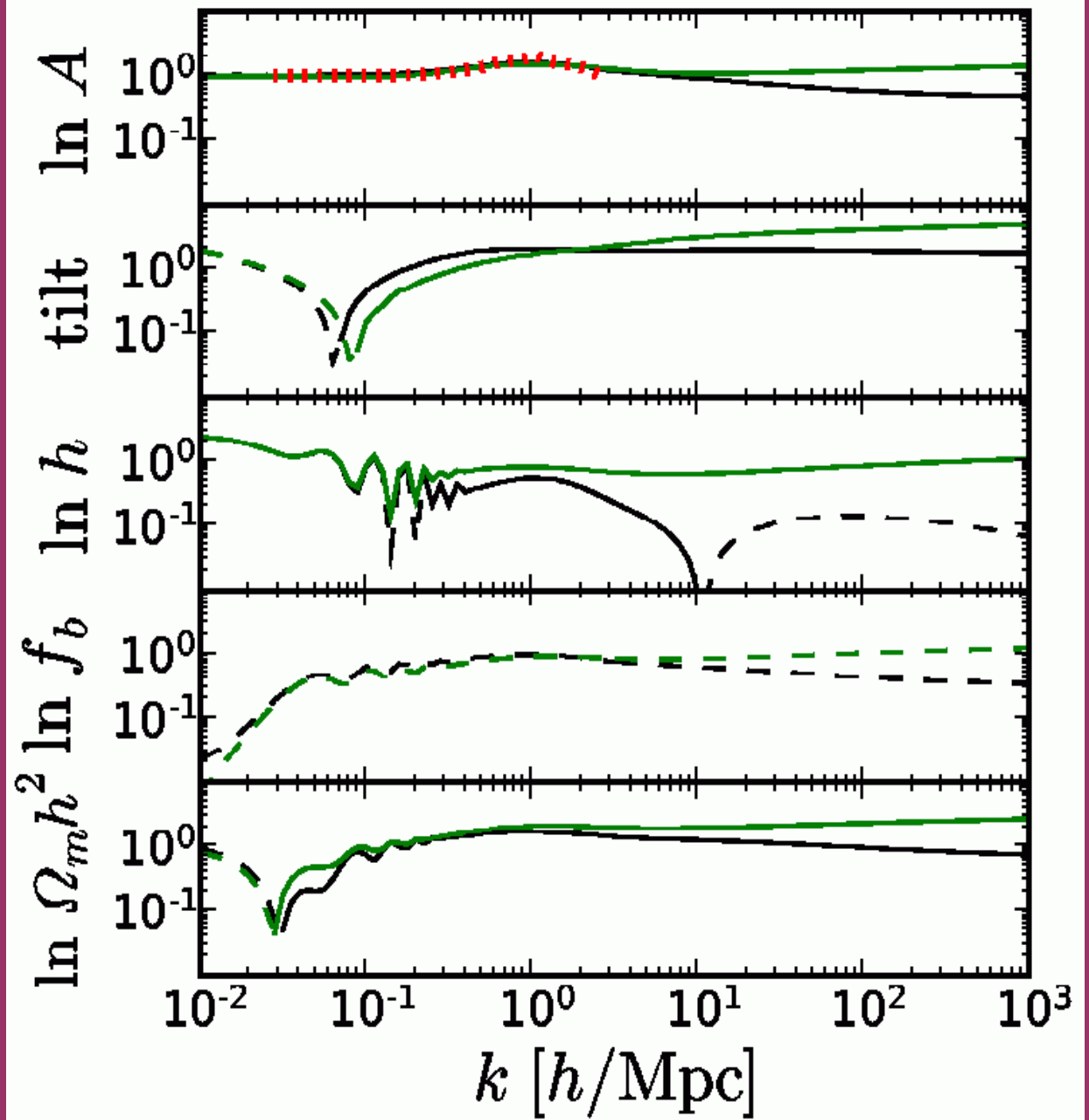
Information curve comparison:

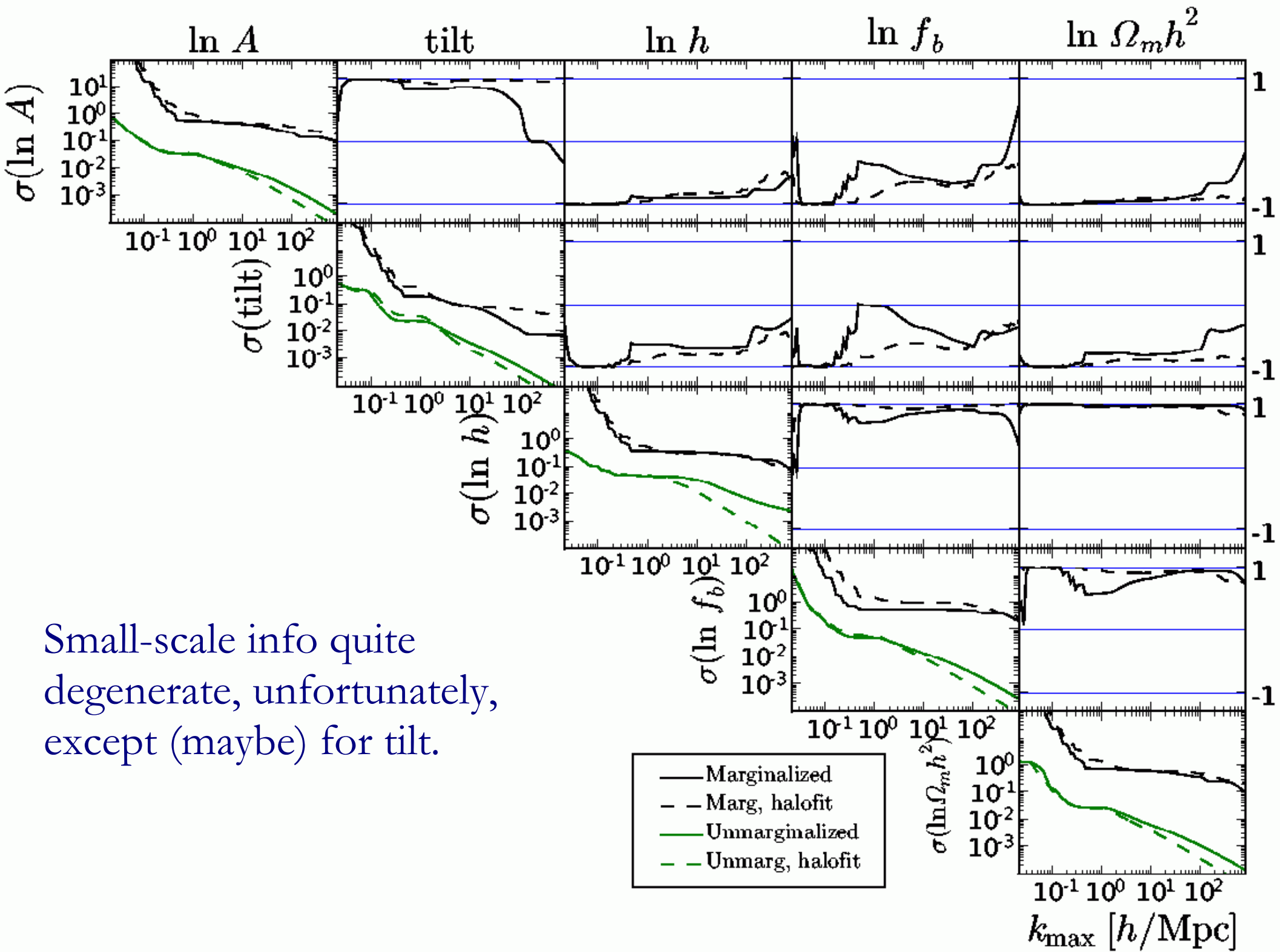
- Same behavior as in simulations.
- Hope for cosmological parameter constraints on small scales? Small-scale info encoded in halo distribution?
- What about multiple parameters?



To investigate degeneracy of parameters, we need derivatives of $P(k)$ wrt some parameters:

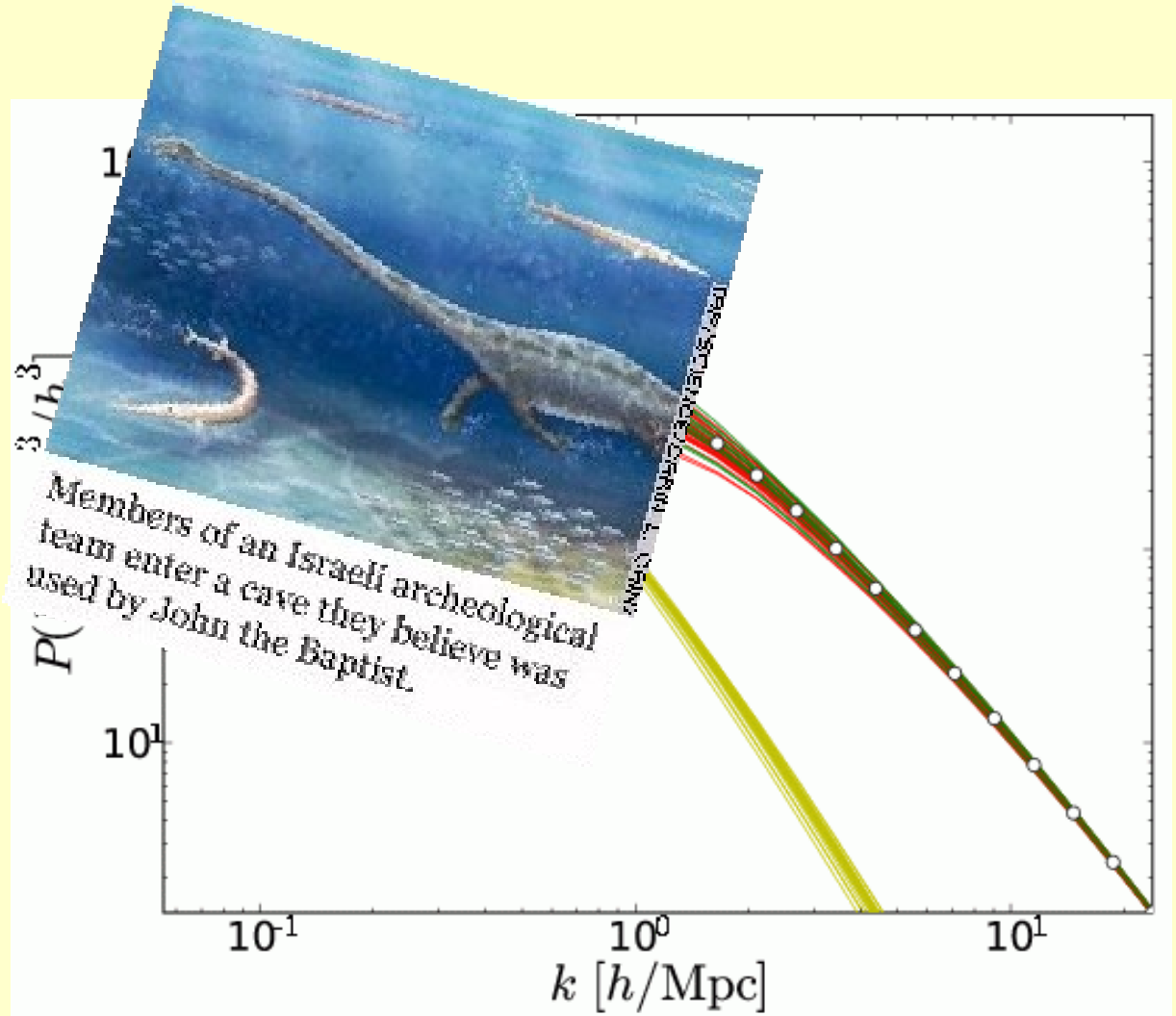
-Tried two methods: our halo model code, **HALOFIT**





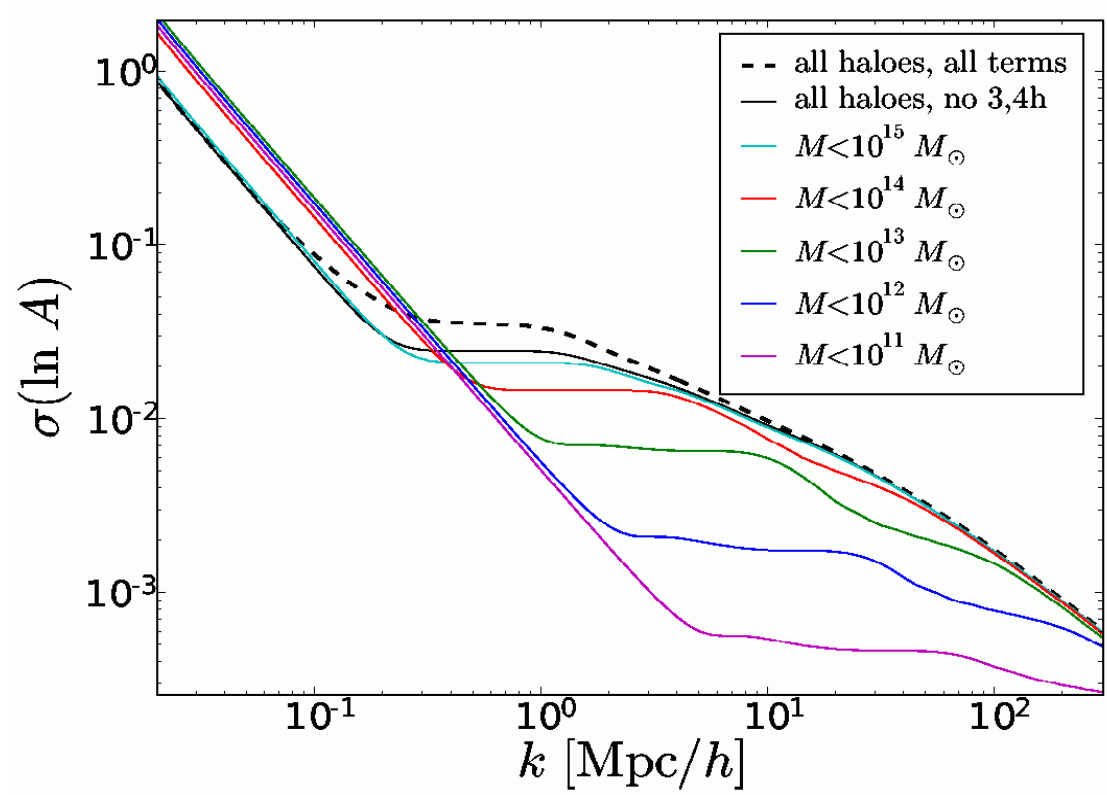
Small-scale info quite degenerate, unfortunately, except (maybe) for tilt.

Physical explanation:
halo number
fluctuations in a finite
volume?



In a finite volume V , the number of haloes in each mass bin is not $Vn(m)dm$ (likely fractional), but is an integer drawn from a distribution with that mean. Most of the (co)variance in the halo model is from this effect.

Look at the power spectrum of matter in rural areas of the Universe?



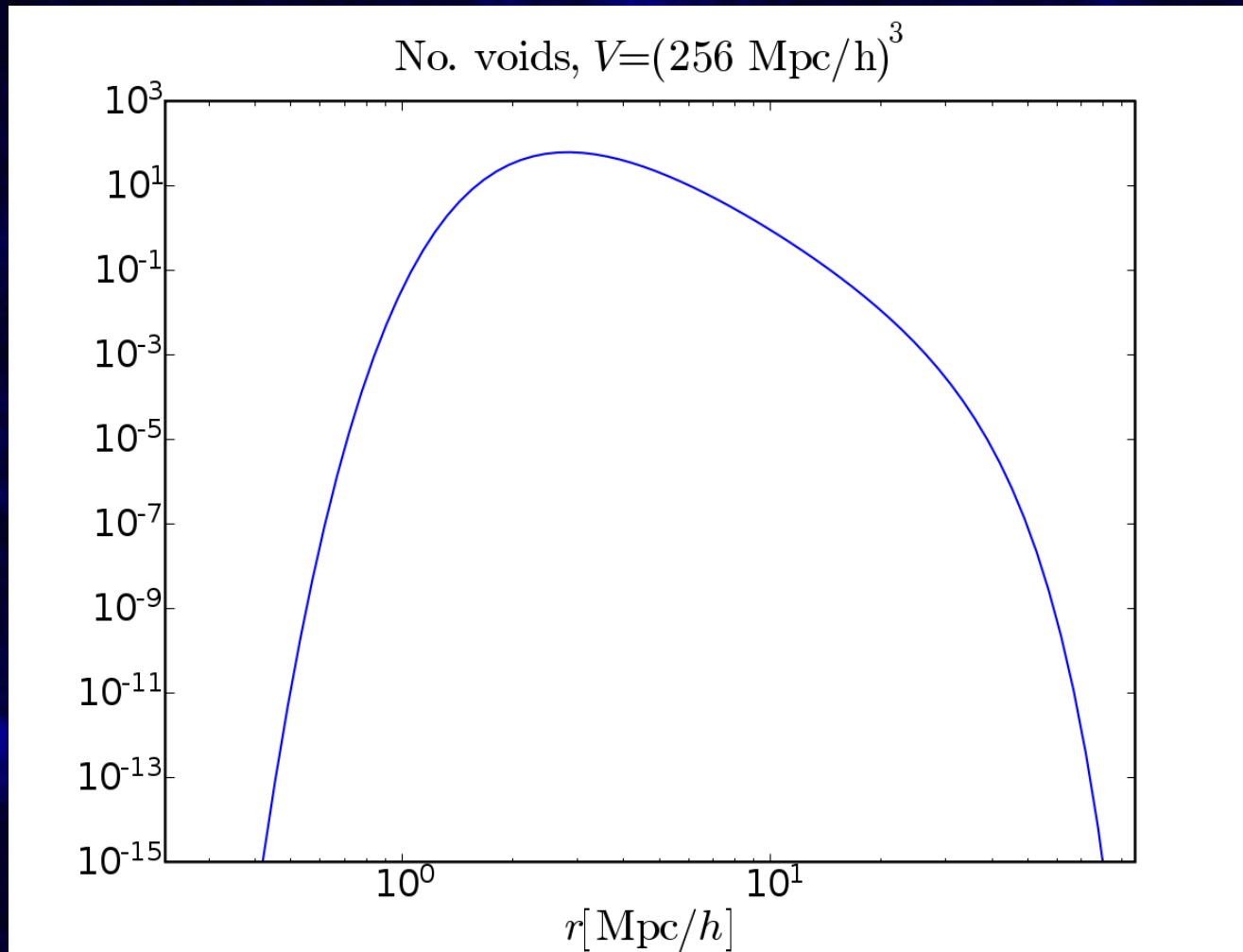
Caveat: need to know halo-halo power spectrum P^{hh} and its covariance; even P^{hh} itself is rather elusive on translinear scales (Smith, Scoccimarro, & Sheth 2006).

If dark matter is collisionless, cosmological info should not be lost. Possibilities:

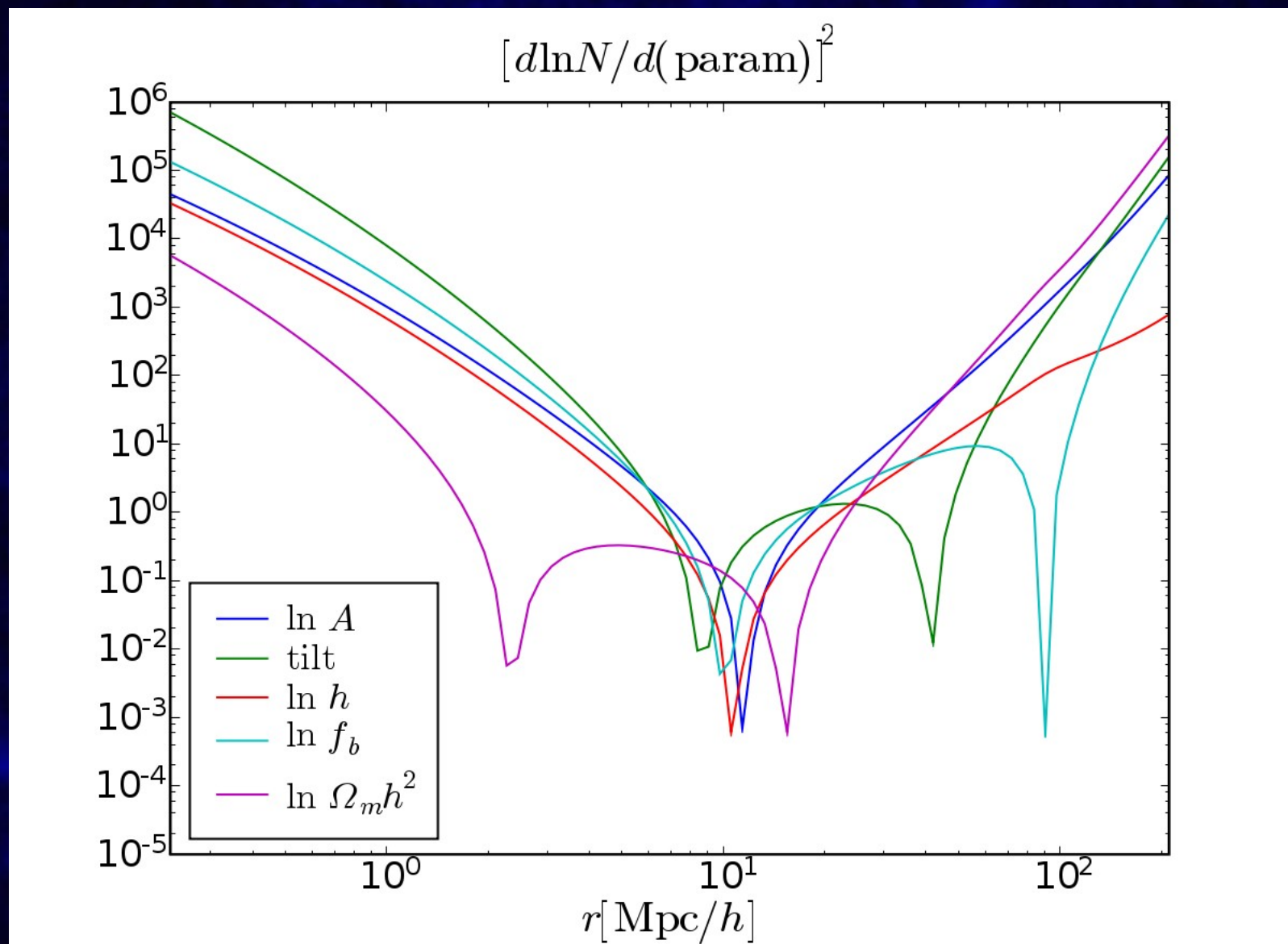
- Info is transferred to smaller scales in the power spectrum
- Info is transferred to low higher-order statistics
- Info is transferred to statistics involving all higher-order statistics, e.g. void stats (our eyes perceive information there)
- Chaotic system \Rightarrow info is there, but virtually impossible to extract.

Try voids (preliminary)

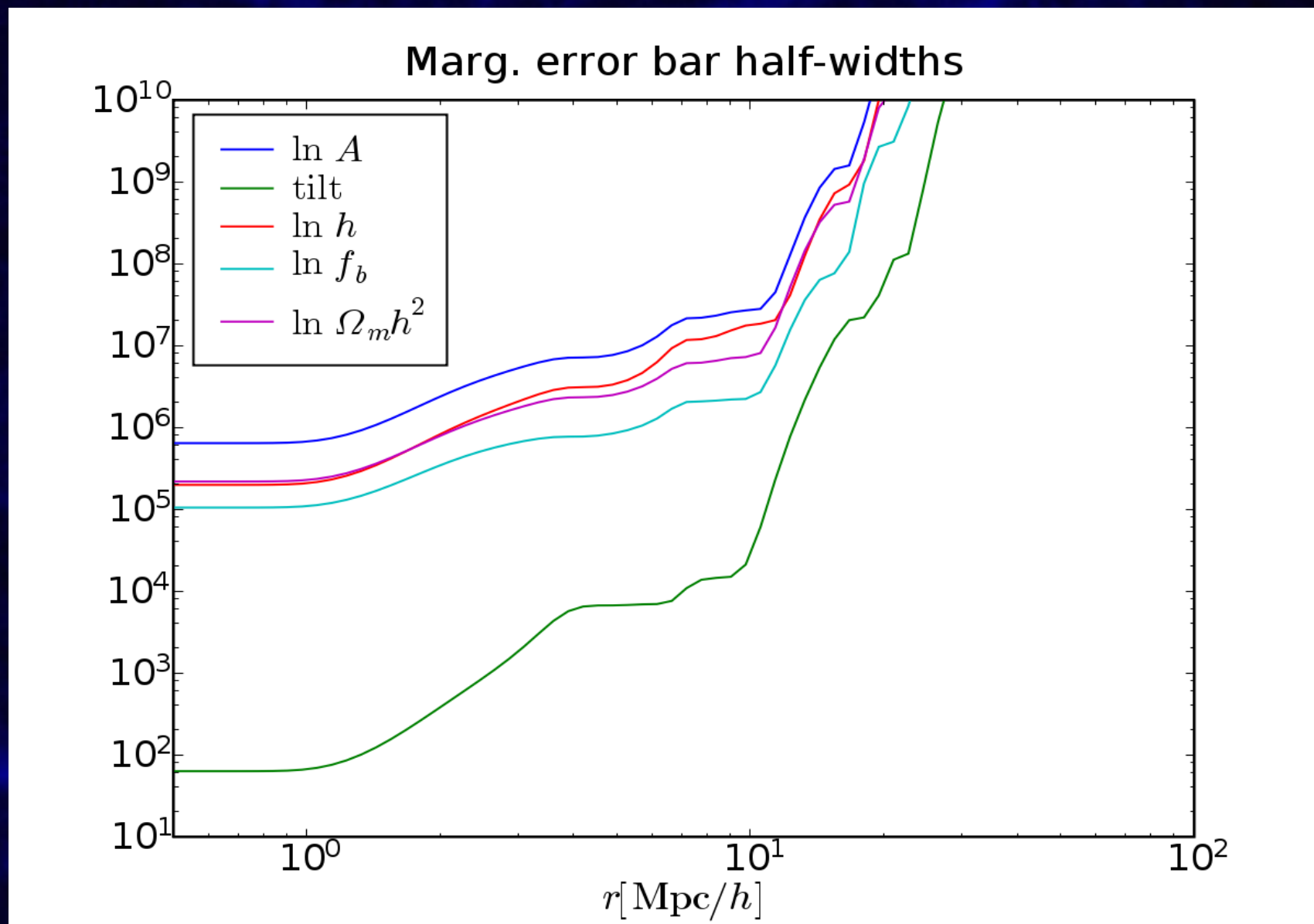
Sheth & van de Weygaert: void mass spectrum has small-scale cut-off. (probably not observable; oh well)



Some dependence of void radius function on cosmological parameters:



Marginalized; not so good.



Conclusions

- The matter power spectrum on small, non-linear scales is disappointing from the point of view of statistical cosmological parameter estimation, even if you can measure and model it well.
- Hope: measure matter, galaxy power spectrum outside of clusters? Use halo (cluster?) power spectrum itself?
- Information in other statistics, more sensitive to non-linearity?

Physical explanation: halo number fluctuations in a finite volume?

It can be proven that the covariance from the 1h term of the power spectrum is the 1h halo-model covariance term.

This term dominates the halo-model covariance.



Why? Consider a product of integrals over Poisson-fluctuated mass functions in a volume V . It can be shown analytically that

$$\langle \int f(m) \tilde{n}(m) dm \int g(m) \tilde{n}(m) dm \rangle = \int f(m) n(m) \int f(m) n(m) dm + \frac{1}{V} \int f(m) g(m) n(m) dm$$

Maybe alarming, since the 2h term of the power spectrum contains such a product. However, the $1/V$ term comes from considering the same halo twice. Something from just one halo cannot enter the 2h term, but it can enter the product of 1h terms.

